# 4.1 Further Differentiation

## Question Paper

| | |
|---|---|
| Course | CIE AS Maths |
| Section | 4. Differentiation |
| Topic | 4.1 Further Differentiation |
| Difficulty | Very Hard |

**Time allowed:** 50

**Score:** /34

**Percentage:** /100

**Question 1**

Use an appropriate method to differentiate each of the following.

(i) $\tan 3x + e^{7-2x^2}$

(ii) $(x^2 + 2x - 8)\cos(3 - x)$

(iii) $\dfrac{\ln 7x}{\sin(x^2+5)}$

(iv) $\sqrt{\cos 4x}$

**[8 marks]**

## Question 2

A curve has the equation $y = 3^x + 2^{-x}$.

Show that the gradient of the normal to the curve at the point $\left(1, \frac{7}{2}\right)$ is

$$\frac{2}{\ln 2 - 6 \ln 3}$$

[4 marks]

## Question 3

Find the derivative of the function $f(x) = \sin\left(\cos\left(\ln\frac{1}{x}\right)\right),\ x > 0$.

[4 marks]

**Question 4a**

(a) Show that the derivative $y = 4^{-x^4}$ is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -(\ln 4)x^3 4^{1-x^4}$$

**[4 marks]**

**Question 4b**

(b) Hence find the equation of the tangent to the curve at the point $\left(1, \frac{1}{4}\right)$, giving your answer in the form $y = ax + b$, where $a$ and $b$ are to be given as exact values.

**[2 marks]**

**Question 5a**

Differentiate with respect to $x$, simplifying your answers where possible:

(a) $(5 + \sin^2 3x)e^{x^2-3x+2}$

**[3 marks]**

**Question 5b**

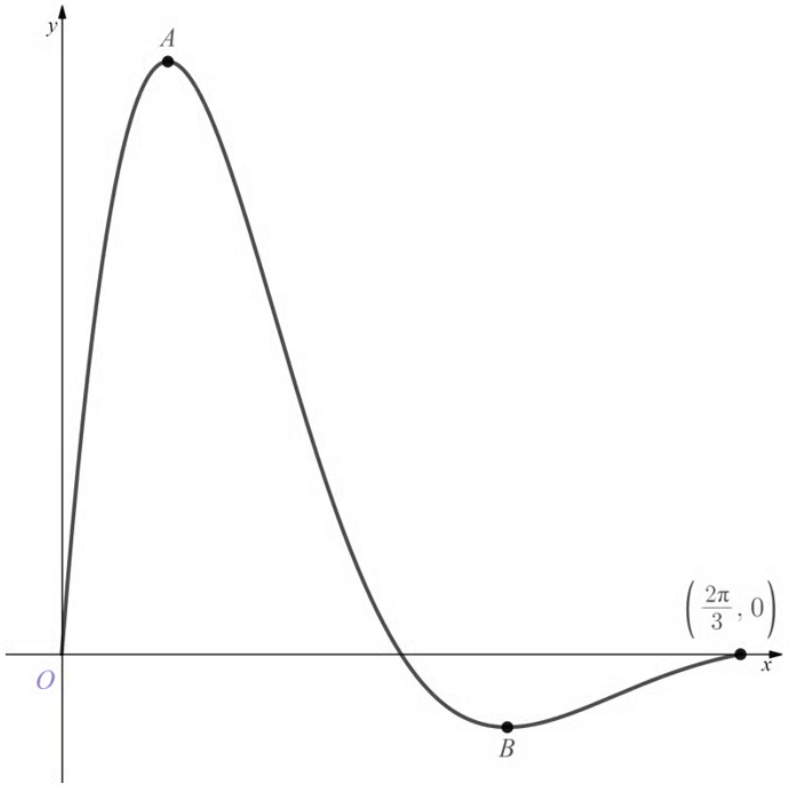
(b) $3^{\sqrt{x}}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$

**[3 marks]**

**Question 6**

The diagram below shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = \frac{\sin 3x}{e^{2x-3}}, \qquad 0 \leq x \leq \frac{2\pi}{3}$$




The points $A$ and $B$ are maximum and minimum points, respectively.

Find the range of $\mathrm{f}(x)$, giving your answer correct to 3 decimal places.

**[6 marks]**